Quantum Coherence Conservation by Growth in Environmental Dissipation Rate

Akira SaiToh [a,*] Robabeh Rahimi [b] Mikio Nakahara [b]

[a] *Department of Systems Innovation, Graduate School of Engineering Science, Osaka University, Toyonaka, Osaka 560-8531, Japan*
[b] *Department of Physics, Kinki University, Higashi-Osaka, Osaka 577-8502, Japan*

Abstract

Quantum coherence conservation is shown to be achieved by a very high rate of dissipation of an environmental system coupled with a principal system. This effect is not in the list of previously-known strategies of noise suppression, such as Zeno effect, dynamical decoupling, quantum error correction code, and decoherence free subspace. An analytical solution is found for a simplified model of a single qubit coupled with an environmental single qubit dissipating rapidly. We also show examples of coherence conservation in a spin-boson linear coupling model with a numerical evaluation.

Key words: Decoherence suppression, dissipative environment, entanglement
PACS: 03.65.Yz, 03.67.Pp, 03.67.-a

Suppressing decoherence in quantum systems is of growing interest in the light of the rapid development of quantum computing [1,2]. There have been many schemes proposed for this purpose in several categories, namely, Zeno effect [3], dynamical decoupling (or bang-bang control) [4,5], quantum error correction code (QECC) [6,7,8], and decoherence free subspace (DFS) [9,10,11]. Conventional schemes for suppressing decoherence focus on controlling a principal system that is under influence of environmental systems. The quantum Zeno effect uses a sequence of projective operations (usually projective measurements) mapping states onto subspaces of a state space [12]. A quantum state is kept inside of a subspace if projective operations are applied very frequently before noise kicks the state out of the subspace. Dynamical decoupling has been studied intensively in the field of nuclear magnetic resonance (NMR) from the dawn of the field (see, e.g., Ref. [13]) and later extended to other physical systems [4,5,14,15,16]. This scheme uses a train of regular short pulses applied to a principal system to cancel time evolutions under noise. The concept of QECC is to discard a subspace easily affected by noise. A QECC uses a space of code words generated from original states by adding a certain redundancy so that a recovery from corruption is possible. The DFS scheme utilizes a subspace of states that are unchanged by given noise operators. Quantum computation is performed in the subspace.

Studies on coherence conservation so far are mostly based on the assumption that we do not have control over environment. There may be alternatives available if we focus on the idea of controlling environmental parameters. A method to control effective coupling using a dynamical control field in the presence of time-dependent external field was recently reported by Jirari and Pötz [17,18]. We pursue the possibility of passive control, rather than dynamical control, in a simple picture of decoherence (See Ref. [19] for conventional decoherence models).

There are many parameters affecting the decoherence factor in realistic models. It is often mentioned that there are the strong coupling regime and the weak coupling regime for a system consisting of a principal system and a noise source in general. (A typical case is a qubit under a random telegraphic noise [20].) The two regimes involve significantly different dependencies of the decoherence factor on the parameters describing noise. We need to choose a proper parameter with which one can alter the behaviour of the decoherence factor for our purpose.

We report in this letter coherence conservation achieved by a very high rate of dissipation of an environmental system coupled with a principal system. A highly dissipative environmental system is wiped out before affecting the principal system. A clue of the phenomenon was originally found in our numerical simulation of bang-bang control of entanglement in a spin-bus

* Corresponding author.
Email addresses: saitoh@qc.ee.es.osaka-u.ac.jp (Akira SaiToh), rahimi@alice.math.kindai.ac.jp (Robabeh Rahimi), nakahara@math.kindai.ac.jp (Mikio Nakahara).

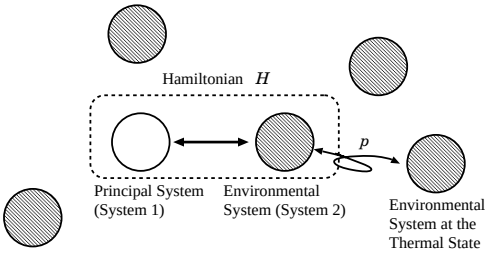

Fig. 1. Model for the system consisting of the principal system (system 1) and the environmental system (system 2) whose time evolution is governed by the Hamiltonian H. System 2 is replaced with a thermal environmental system with the dissipation probability p for the time interval τ.

model [21]. We investigate the phenomenon in detail by using a mathematical analysis and a numerical simulation for simplified models. An analytical solution is found for a model of a single qubit coupled with an environmental single qubit dissipating rapidly. The effect is also found in a spin-boson linear coupling model by using a numerical evaluation.

Let us assume that the entire environment is so large that the environmental system (system 2) coupled with the principal system (system 1) is a part of a large environment and hence it is replaced with a thermal environmental system with probability p (namely, with some dissipation rate) per certain time interval τ. Systems 1 and 2 are represented by the density matrix $\rho^{[1,2]}$; a thermal environmental system is represented by the density matrix σ. The Hamiltonian affecting the time evolution is reduced to the one consisting only of the time-independent Hamiltonian H that governs systems 1 and 2 including their interaction. This model is illustrated in Fig. 1. For a small time interval Δt, the evolution of the systems 1 and 2 obeys the equation

$$\rho^{[1,2]}(\tilde{t}+\Delta t) = e^{-iH\Delta t}\Big[x^{\Delta t}\rho^{[1,2]}(\tilde{t}) + (1-x^{\Delta t})\mathrm{Tr}_2\rho^{[1,2]}(\tilde{t})\otimes\sigma\Big]e^{iH\Delta t}, \quad (1)$$

where $x = (1-p)^{1/\tau}$ and $\tilde{t}$ denotes a certain time step. The dissipation rate p can be modified by changing the experimental setup under a static control.

Let us begin with an analytical evaluation of this model in the case where the systems 1 and 2 are single-qubit systems. For further simplicity, we impose the following conditions. The principal system is originally represented by a density matrix

$$\rho^{[1]}(0) = \begin{pmatrix} a & b \\ b^* & 1-a \end{pmatrix}$$

with $0 \le a \le 1$ and $0 \le |b| \le \sqrt{a(1-a)}$. The environmental system at thermal equilibrium is represented by the maximally-mixed density matrix

$$\rho^{[2]}(0) = \sigma = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}.$$

The initial state of the total system is set to $\rho^{[1,2]}(0) = \rho^{[1]}(0)\otimes\rho^{[2]}(0)$. The Hamiltonian H is set to $cI_z\otimes I_z = \mathrm{diag}(c/4,-c/4,-c/4,c/4)$ [here, $I_z = \mathrm{diag}(1/2,-1/2)$].

With these simplifications, one can find the form of the density matrix at time $t = m\Delta t$ ($m \in \{0,1,2,\ldots\}$) as

$$\rho^{[1,2]}(m\Delta t) = \begin{pmatrix} a/2 & 0 & f_m & 0 \\ 0 & a/2 & 0 & g_m \\ f_m^* & 0 & (1-a)/2 & 0 \\ 0 & g_m^* & 0 & (1-a)/2 \end{pmatrix},$$

with functions f_m and g_m depending on m. The functions f_m and g_m obey the system of recurrence formulae:

$$\begin{cases} f_{m+1} = \dfrac{1}{2}e^{-ic\Delta t/2}\left[f_m + g_m + x^{\Delta t}(f_m - g_m)\right] \\ g_{m+1} = \dfrac{1}{2}e^{ic\Delta t/2}\left[f_m + g_m - x^{\Delta t}(f_m - g_m)\right] \end{cases}$$

with $f_0 = g_0 = b/2$. This leads to the following recurrence formula:

$$\kappa_{m+2} = (1+x^{\Delta t})\cos(c\Delta t/2)\kappa_{m+1} - x^{\Delta t}\kappa_m, \quad (2)$$

where $\kappa_m = f_m$ or g_m with $f_0 = g_0 = b/2$, $f_1 = be^{-ic\Delta t/2}/2$, and $g_1 = be^{ic\Delta t/2}/2$.

One can derive functions $f(t) = \lim_{\Delta t\to 0, m\Delta t = t} f_m$ and $g(t) = \lim_{\Delta t\to 0, m\Delta t = t} g_m$ in the following way. By linearlization, Eq. (2) is put in the form:

$$\kappa_{m+2} - 2\kappa_{m+1} + \kappa_m - \Delta t \ln x(\kappa_{m+1} - \kappa_m) + (\Delta t)^2\frac{c^2}{4}\kappa_{m+1} - \frac{(\Delta t)^2}{2}(\ln x)^2(\kappa_{m+1} - \kappa_m) + \mathcal{O}[(\Delta t)^3] = 0$$

Dividing this equation by $(\Delta t)^2$ and taking the limit $\Delta t \to 0$ lead to

$$\partial^2\kappa(t)/\partial t^2 - \ln x\ \partial\kappa(t)/\partial t + c^2\kappa(t)/4 = 0,$$

where $\kappa(t) = \lim_{\Delta t\to 0, m\Delta t = t}\kappa_m$. The solution of this differential equation is

$$\kappa(t) = u_\kappa e^{-r_+ t} + v_\kappa e^{-r_- t}$$

with constants u_κ and v_κ ($\kappa = f$ or g), and the complex decoherence factor

$$r_\pm = -\left[\ln x \pm \sqrt{(\ln x)^2 - c^2}\right]/2.$$

The real part of $r_\pm$ is nonnegative because $\ln x = \ln(1-p)/\tau \le 0$ and $|\ln x| \ge |\mathrm{Re}\sqrt{(\ln x)^2 - c^2}|$. (Here, the square root is positive.)

We need to impose the conditions that $\kappa(0) = b/2$ and $\kappa'(0) = \lim_{\Delta t\to 0}(\kappa_1 - \kappa_0)/\Delta t$. The latter condition can be written as $-r_+u_f - r_-v_f = -ibc/4$ and $-r_+u_g - r_-v_g = ibc/4$. With these conditions, we obtain

$$u_f = \frac{ibc - 2br_-}{4(r_+ - r_-)},\quad v_f = \frac{-ibc + 2br_+}{4(r_+ - r_-)},$$
$$u_g = \frac{-ibc - 2br_-}{4(r_+ - r_-)},\quad v_g = \frac{ibc + 2br_+}{4(r_+ - r_-)}.$$

Consequently, we have

$$f(t) = \frac{ibc - 2br_-}{4(r_+ - r_-)}e^{-r_+ t} + \frac{-ibc + 2br_+}{4(r_+ - r_-)}e^{-r_- t},$$

$$g(t) = \frac{-ibc - 2br_-}{4(r_+ - r_-)} e^{-r_+ t} + \frac{ibc + 2br_+}{4(r_+ - r_-)} e^{-r_- t}.$$

One can now write the reduced density matrix of the principal system at t as

$$\rho^{[1]}(t) = \begin{pmatrix} a & \eta(t) \\ \eta(t)^* & 1-a \end{pmatrix}$$

with

$$\eta(t) = b \left(\frac{-r_-}{r_+ - r_-} e^{-r_+ t} + \frac{r_+}{r_+ - r_-} e^{-r_- t} \right).$$

Let us investigate $r_\pm$ in details in relation to p. One obvious fact is that exponential decay is caused by the real part of $e^{-r_\pm t}$. Thus we focus on the behaviour of $\mathrm{Re}\, r_\pm$. We find that $\mathrm{Re}\, r_+$ increases as $-(\ln x)/2$ when $0 \le p \le 1 - e^{-c\tau}$ and decreases with convergence to zero when $1 - e^{-c\tau} < p \le 1$. In contrast, $\mathrm{Re}\, r_-$ increases as $-(\ln x)/2$ when $0 \le p \le 1 - e^{-c\tau}$ and increases more rapidly when $1 - e^{-c\tau} < p \le 1$. This is clearly depicted in Fig. 2. This result suggests that the

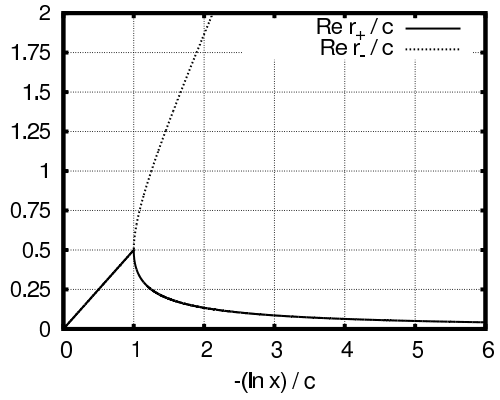

Fig. 2. Plot of $\mathrm{Re}\, r_\pm / c$ as functions of $-(\ln x)/c$. The decoherence factors $\mathrm{Re}\, r_\pm$ increases until they reaches $c/2$ as the dissipation rate increases. The factor $\mathrm{Re}\, r_+$ starts decreasing at the point of $-\ln x = c$ (i.e., $p = 1 - e^{-c\tau}$) while the factor $\mathrm{Re}\, r_-$ starts increasing rapidly at this point.

decoherence factor $\mathrm{Re}\, r_+$ is small for a large dissipation rate $p > 1 - e^{-c\tau}$. In addition, $\mathrm{Re}\, e^{-r_-}$ rapidly converges to 0 for $p > 1 - e^{-c\tau}$. Thus the dominant term for such a large p is $b\frac{-r_-}{r_+ - r_-} e^{-r_+ t}$, which converges to b. This phenomenon can be understood physically: the environmental system is wiped out quickly before absorbing coherence information of the principal system as p approaches to unity. Therefore this effect may be called *quantum wipe effect*.

In addition, the behaviour of $\mathrm{Im}\, r_\pm$ is rather simple, as shown in Fig. 3. They vanish for $p \ge 1 - e^{-c\tau}$; this suggests that we do not observe oscillation in $\eta(t)$ for such a large p.

As an example, we will see the time evolution of $|\eta(t)|$ for $\tau = 1.0 \times 10^{-3}$s and $c = 1.0 \times 10^3$Hz. Figure 4 shows clear suppression of decoherence for small p and also for large p. Suppression of oscillation is found for large p.

The phenomenon we have seen above can be found also in a spin-boson linear coupling model. Let us consider a system consisting of a single spin-1/2 (system 1) and a mode of bosons (system 2) coupled with the spin system. We assume that the mode of the bosons is in resonance with the precession of

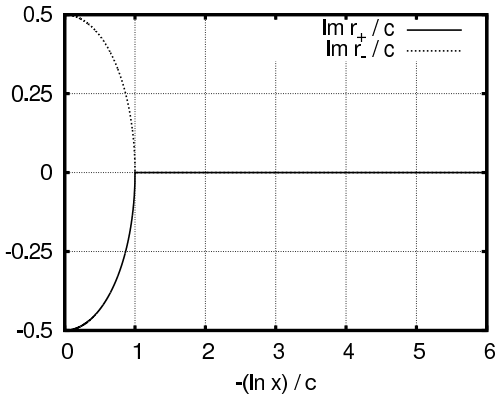

Fig. 3. Plot of $\mathrm{Im}\, r_\pm / c$ as functions of $-(\ln x)/c$. These imaginary factors vanish for $-\ln x \ge c$ (i.e., $p \ge 1 - e^{-c\tau}$).

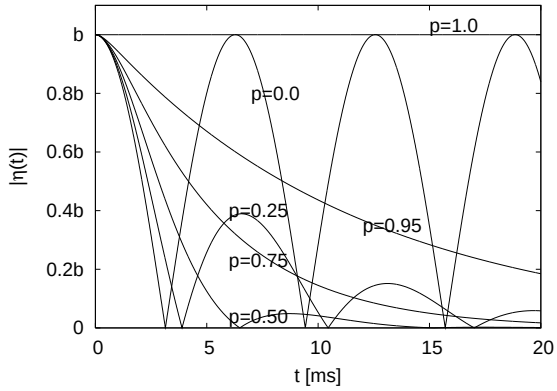

Fig. 4. Time evolution of $|\eta(t)|$ for several different values of p (0.0, 0.25, 0.5, 0.75, 0.95, and 1.0) when $\tau = 1.0 \times 10^{-3}$s and $c = 1.0 \times 10^3$Hz.

the spin system (the resonance frequency is ν). Assuming that systems 1 and 2 are surrounded by a bath of bosons of a cavity and/or a sample holder, we employ the same model illustrated in Fig. 1.

Let us set the initial state of the spin to

$$\rho^{[1]}(0) = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}.$$

The initial state of the bosonic system is $\rho^{[2]}(0) = e^{-\beta H^{[2]}}/Z$ where β is defined as $\beta = (k_\mathrm{B} T)^{-1}$ (k_B is the Boltzmann constant and T is temperature), $H^{[2]} = \nu a^\dagger a$ is the bosonic system Hamiltonian ($a^\dagger$ and a are the creation and annihilation operators), and Z is the partition function. The thermal state probabilistically replacing $\rho^{[2]}$ is $\sigma = \rho^{[2]}(0)$. The initial state of the total system is set to $\rho^{[1,2]}(0) = \rho^{[1]}(0) \otimes \rho^{[2]}(0)$. Let the full Hamiltonian be $H = H^{[1]} + H^{[2]} + H_c$ with the spin-system Hamiltonian $H^{[1]} = \nu I_z$ and the coupling Hamiltonian $H_c = c I_z (a^\dagger + a)$ (here, c is a coupling constant).

A numerical simulation is used to compute the time evolution governed by Eq. (1). We have taken the parameters: $\nu = 3.4 \times 10^{10}$Hz, $T = 1.0$mK, $c = 1.0 \times 10^7$Hz, and $\tau = 1.0 \times 10^{-8}$s. The values of ν and T are taken from physically available values for an electron spin coupled with an on-resonance bosonic mode (a cavity mode and/or a phonon mode of a solid sample) in a

low-temperature Q-band electron-nuclear double resonance (Q-band ENDOR) system. In simulations, Δt is set to 5.0×10^{-10}s.

The simulation showed a clear suppression of decoherence in $|\langle 0|\rho^{[1]}(t)|1\rangle| = |\langle 0|\mathrm{Tr}_2\rho^{[1,2]}(t)|1\rangle|$ for large p as shown in Fig. 5.

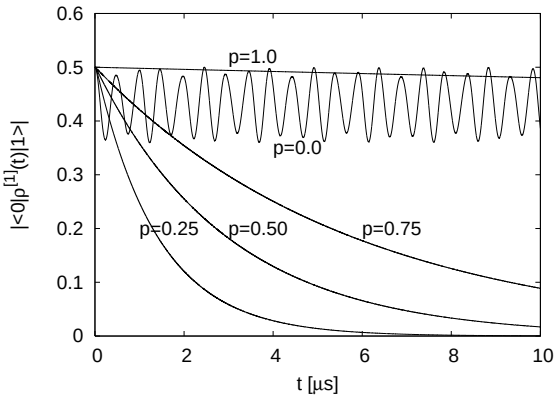

Fig. 5. Numerically computed time evolution of $|\langle 0|\rho^{[1]}(t)|1\rangle|$ for several different values of p.

For the next example, we numerically evaluate the decay of entanglement of a principal system (system 1) consisting of one electron spin and one nuclear spin each of which is coupled with an on-resonance bosonic mode (system 2). The measure of entanglement that we use here is negativity [22,23] [1] . The evolution of the systems is governed by Eq. (1). Let us consider the following Hamiltonians (the subscripts 0 and 1 denote labels assigned to individual spins and corresponding on-resonance bosonic modes). (i) The Hamiltonian of the principal system: $H^{[1]} = \nu_0 I_{z0} + \nu_1 I_{z1} + A_{01} I_{z0} I_{z1}$ with the precession frequencies ν_0, ν_1 and the spin-spin coupling constant A_{01} (here, I_{zi} is an I_z operator acting on the ith spin). (ii) The Hamiltonian of the bosonic system: $H^{[2]} = \nu_0 a_0^\dagger a_0 + \nu_1 a_1^\dagger a_1$. (iii) The Hamiltonian of spin-boson couplings: $H_c = c_0 I_{z0}(a_0^\dagger + a_0) + c_1 I_{z1}(a_1^\dagger + a_1)$ with constants c_0 and c_1. The total Hamiltonian is $H = H^{[1]} + H^{[2]} + H_c$.

The initial reduced density matrix of the principal system is set to

$$\rho^{[1]}(0) = \frac{|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|}{2}$$

and that of the bosonic system is set to the thermal one $\rho^{[2]}(0) = \sigma = e^{-\beta H^{[2]}}/Z$. We set $\rho^{[1,2]}(0) = \rho^{[1]}(0) \otimes \rho^{[2]}(0)$. The following constants are used: $\nu_0 = 3.4\times10^{10}$Hz, $\nu_1 = 4.87\times10^7$Hz, $A_{01} = 1.0\times10^7$Hz, $T = 1.0\times10^{-3}$mK, $c_0 = c_1 = 1.0\times10^7$Hz, and $\tau = 1.0\times10^{-8}$s. Here, the values of ν_0, ν_1, and T are chosen by considering an electron spin and a nuclear spin coupled with on-resonance bosonic modes in a Q-band ENDOR system. A numerical simulation is performed with $\Delta t = 5.0\times10^{-10}$s.

[1] Negativity $\mathcal{N}$ can be calculated for a density matrix $\rho^{[a,b]}$ of a bipartite system consisting of subsystems a and b in the following way: $\mathcal{N}(\rho^{[a,b]}) = [\|(I\otimes\Lambda_{\mathrm{T}})\rho^{[a,b]}\| - 1]/2$ where Λ_{T} is the transposition map acting on the subsystem b and hence $I\otimes\Lambda_{\mathrm{T}}$ is a partial transposition map.

We show the time evolution of negativity of the principal system in Fig. 6. An improvement of the entanglement conservation is found for large p and it is especially conspicuous for $p \geq 0.95$.

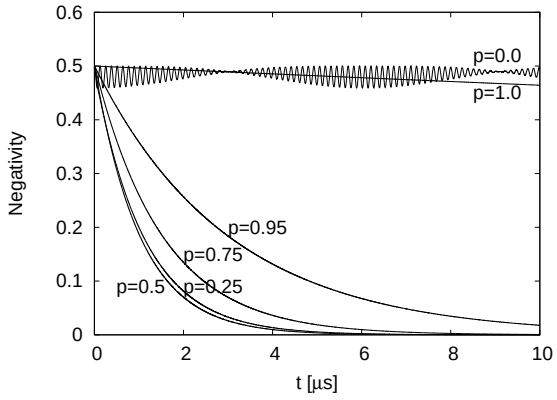

Fig. 6. Numerically computed time evolution of negativity of the principal system for several values of p.

In summary, we considered a model in which a principal system is coupled with an environmental system that is probabilistically replaced with a thermal environmental system. In such a model, a high dissipation rate of the environmental system results in a conservation of coherence of the principal system. This phenomenon may be called quantum wipe effect. We have shown that this phenomenon is found in a simple model of qubit-qubit coupling by an analytical calculation. An increase of a dissipation rate over a certain threshold value results in coherence conservation while an increase of the dissipation rate otherwise results in larger decoherence, as shown in Fig. 2. This is reminiscent of the relation between the Zeno effect and the Anti-Zeno effect [24], namely that very quick applications of projective operations are effective for coherence conservation while slow applications of them result in larger decoherence. The quantum wipe effect has been also found numerically for a model of spin-boson linear coupling.

The usefulness of the quantum wipe effect is dependent on how small the threshold value of the dissipation rate is. A coupling constant is required to be small to obtain a small threshold value. It is thus expected that this effect will be verified experimentally in future with a weak system-environment coupling.

AS is supported by the JSPS Research Fellowship for Young Scientists. RR is supported by the Sasakawa Scientific Research Grant from JSS. MN would like to thank for a partial support of the Grant-in-Aid for Scientific Research from JSPS (Grant No. 19540422). This work is partially supported by "Open Research Center" Project for Private Universities: matching fund subsidy from MEXT.

References

[1] J. Gruska, Quantum Computing, McGraw-Hill, London, 1999.
[2] M.A. Nielsen and I.L. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, Cambridge, 2000.

[3] B. Misra and E.C.G. Sudarshan, J. Math. Phys. 18 (1977) 756.
[4] M. Ban, J. Mod. Opt. 45 (1998) 2315.
[5] L. Viola and S. Lloyd, Phys. Rev. A 58 (1998) 2733.
[6] P.W. Shor, Phys. Rev. A 52 (1995) 2493(R).
[7] D. Gottesman, Phys. Rev. A 54 (1996) 1862.
[8] R. Laflamme, C. Miquel, J.P. Paz, and W.H. Zurek, Phys. Rev. Lett. 77 (1996) 198.
[9] P. Zanardi and M. Rasetti, Phys. Rev. Lett. 79 (1997) 3306.
[10] L.-M. Duan and G.-C. Guo, Phys. Rev. Lett. 79 (1997) 1953.
[11] D.A. Lidar, I.L. Chuang, and K.B. Whaley, Phys. Rev. Lett. 81 (1998) 2594.
[12] P. Facchi and S. Pascazio, Phys. Rev. Lett. 89 (2002) 080401.
[13] R.R. Ernst, G. Bodenhausen, and A. Wokaun, Principles of Nuclear Magnetic Resonance in One And Two Dimensions, Oxford University Press, Oxford, 1987.
[14] D. Vitali and P. Tombesi, Phys. Rev. A 59 (1999) 4178.
[15] L.-A. Wu, and D.A. Lidar, Phys. Rev. A 70 (2004) 062310.
[16] M. Sasaki, A. Hasegawa, J. Ishi-Hayase, Y. Mitsumori, and F. Minami, Phys. Rev. B 71 (2005) 165314.
[17] H. Jirari and W. Pötz, Phys. Rev. A 74 (2006) 022306.
[18] H. Jirari and W. Pötz, Europhys. Lett. 77 (2007) 50005.
[19] W.H. Louisell, Quantum Statistical Properties of Radiation, John Wiley & Sons, New York, 1973.
[20] J. Schriefl, M. Clusel, D. Carpentier, and P. Degiovanni, Phys. Rev. B 72 (2005) 035328.
[21] R. Rahimi, A. SaiToh, and M. Nakahara, accepted for publication in J. Phys. Soc. Japan.
[22] K. Życzkowski, P. Horodecki, A. Sanpera, and M. Lewenstein, Phys. Rev. A 58 (1998) 883.
[23] G. Vidal and R.F. Werner, Phys. Rev. A 65 (2002) 032314.
[24] A.G. Kofman and G. Kurizki, Nature (London) 405 (2000) 546.